The thermal barrier, convection scheme will always use less air than the same no protection, convection scheme. How much more favorable depends "only" on the film coefficient. High pressure ratio engines with attendant high heat fluxes favor a thermal barrier approach.

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Selective Reinforcement of Wing Structure for Flutter Prevention

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Introduction

THE anisotropic properties of filamentary composites permit a high degree of precision in tailoring and modifying stiffness and strength properties in a structure. This capability, coupled with the high strength and/or stiffness to weight ratio, make filamentary composite materials attractive for use in selectively reinforcing primary lifting surfaces of aircraft. This Note presents the results of an analytical study of the application of boron polyimide filamentary composite material to increase the flutter speed of a simple titanium full depth sandwich wing structure designed for strength. The sensitivity of the critical flutter speed to the location of patches of composite bonded to the skin of the wing is investigated.

Mathematical Model of Wing

The dimensions of the wing used in this study are shown in Fig. 1. The wing has a biconvex airfoil composed of variable thickness upper and lower cover plates which carry all the load; the wing is full of fuel. As in Ref. 1, plate theory is used in the structural analysis and piston theory is used in the flutter analysis, but with the structural analysis modified herein to take into account specially orthotropic (i.e., no direct coupling between bending and twisting, Ref. 2) filamentary composite layers on the wing covers, and with wing depth variation effects in the aerodynamic loading neglected.

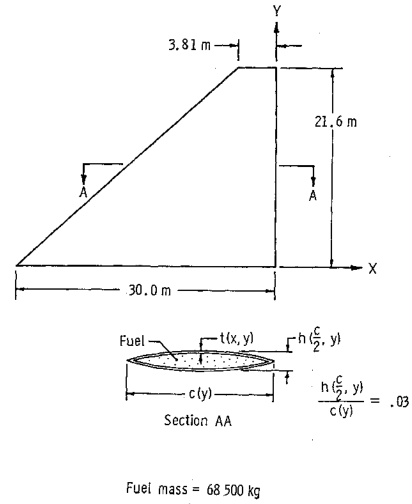

Fig. 1 Wing description.

All-Titanium Wings

Three all-titanium wings were designed to meet, respectively, flutter requirements only, strength requirements only, and both requirements. The wing is to a) be flutter-free at a velocity of 760 m/sec at an altitude of 7620 m ($M = 2.46$); b) support a uniformly distributed loading of 6900 N/m^2. The design process required determining four coefficients c_i in a polynomial representation of the thickness distribution of the wing cover plates

$$t = t_{\min} + \left[c_1\left(1-\frac{y}{y_S}\right)+c_2\left(1-\frac{y}{y_S}\right)^2\right]\left[1-\left(1-\frac{2x}{x_L}\right)\right] + \left[c_3\left(1-\frac{y}{y_S}\right)+c_4\left(1-\frac{y}{y_S}\right)^2\right]\left[1-\left(1-\frac{2x}{x_L}\right)\right] \times \left(1-\frac{2x}{x_L}\right) \quad (1)$$

in which $t_{\min} = 0.051$ cm, is the minimum gage requirement for all designs, x is the chordwise coordinate, y is the spanwise coordinate, x_L is the value of x at the leading edge, and y_S is the semispan. The thickness distribution described by Eq. (1) is minimum gage along the leading and trailing edges and at the tip. Mathematical programming techniques were used to determine values of the design variables c_i which give minimum weight and satisfy the flutter, strength, and minimum gage requirements.

Contour plots showing the cover plate thickness distributions for flutter strength, and combined strength-flutter designs are shown in Fig. 2 along with the mass, flutter speed, and ultimate load for each design. The flutter design meets the flutter requirement, but not the strength requirement. The strength design meets the strength requirement, but does not satisfy the flutter requirement. The strength-flutter design meets both requirements. All designs meet the minimum gage requirement. Because of the large mass of fuel (68,500 kg) carried in the wing, the contribution of the wing covers to the total mass distribution is small but is, nevertheless, included in the flutter analysis used to obtain these results. The primary effect of the thickness distribution is to establish the stiffness distribution.

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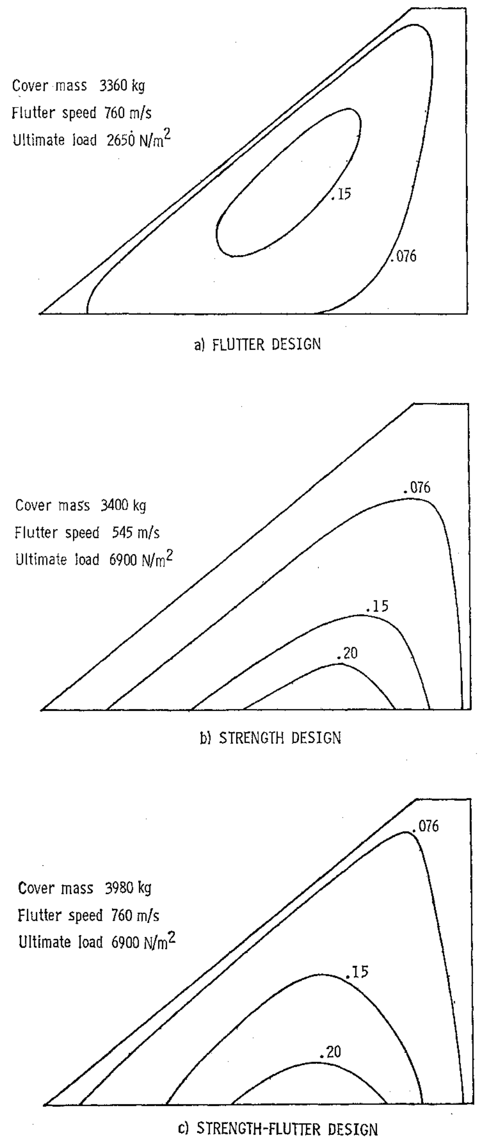

Fig. 2 Contour plots showing all titanium wing cover thickness distribution for flutter, strength, and combined designs obtained using the four design variable thickness distribution given in Eq. (1). Contour labels are in centimeters.

Composite-Stiffened Wings

First, the wing shown in Fig. 2b was investigated to determine the effects of filament orientation on the flutter velocity assuming that two pairs of alternating boron/polyimide lamina at $\pm\theta$ were applied uniformly over the entire surface of the top and bottom cover plates. Figure 3 shows that filament orientations in directions parallel or perpendicular to the flow do not contribute much stiffening for flutter. Maximum stiffening occurred with a fiber orientation θ close to 45°. Although it was recognized that this fiber orientation may not be optimum for patches, 45° was chosen for the fiber orientation of all cases subsequently investigated.

The all-titanium strength design (Fig. 2b) was then stiffened by adding uniform patches of boron/polyimide to the upper and lower wing cover plates in a symmetric fashion about the wing midplane. Calculated flutter speeds at an altitude of 7620 m are presented in Fig. 4 as a function of the mass of material added through patches in each of the four planform regions indicated by the numbers 1, 2, 3, and 4; the intercept of the ordinate represents the flutter speed of the bare strength-designed wing. The dashed line indicates the flutter speed design requirement, and the masses at the intersections of this dashed line with the curves are the mass additions required to achieve this requirement; also shown is the added mass of titanium required in the all-metal design (Fig. 2c). (The tacit assumption is made that the addition of material does not adversely affect the strength of the wing.) The results demonstrate that the change in flutter speed depends strongly on the location of the added composite; the patch in region 3 was most effective and was significantly lighter than the all-

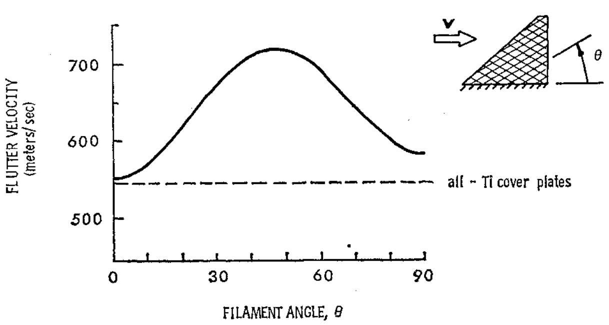

Fig. 3 Effect of fiber orientation on flutter speed with 408 kg of composite reinforcement added uniformly to the wing of Fig. 2b.

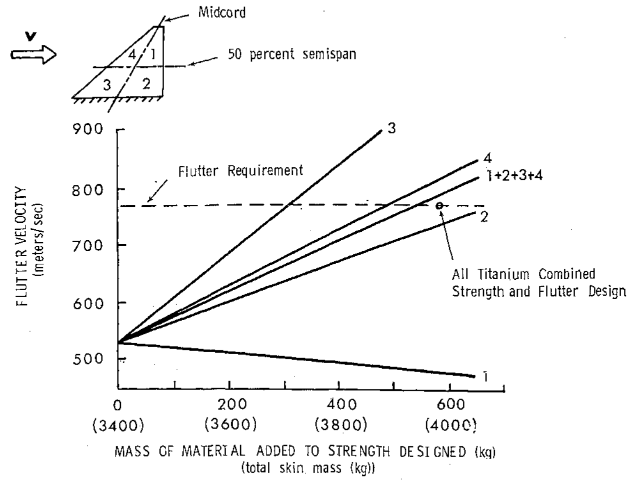

Fig. 4 Addition of Boron/polyimide patches to strength-designed wing.

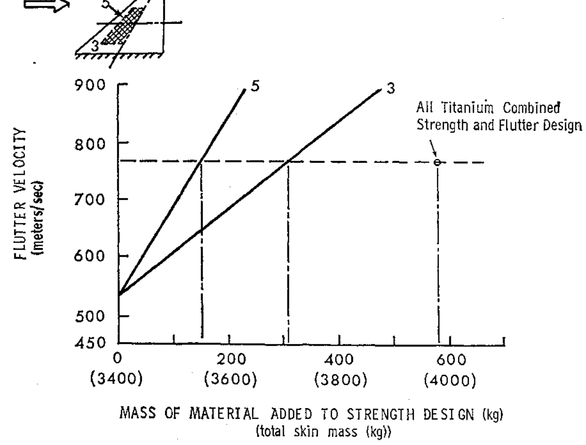

Fig. 5 Effect of selective location of composite patch on flutter of strength-designed wing. Location based on all titanium flutter-designed wing.

metal wing. The addition of composite material in region 1 actually reduced the flutter speed. Flutter calculations using a massless patch showed that the reduction in flutter speed was caused almost entirely by the stiffness of the patch, rather than by the mass of the patch.

The fact that the flutter design in Fig. 2a has a build-up in thickness in a region between the quarter and mid chord suggests that a composite patch covering this region of the strength-designed wing might be more beneficial in reducing the mass addition for flutter prevention than any of the regions considered in Fig. 4. The effects of such a patch, denoted as region 5, is shown in Fig. 5 along with the results from Fig. 4 for a composite patch in region 3 and the minimum-mass all-titanium design. The mass of composite reinforcement required in region 5 is half that required in region 3 and less than one-third the mass addition required by the all-titanium design, even though no optimality criteria or search techniques were used to determine the location or size of the region or the orientation of the fibers; thus even lighter designs may be possible.

These results clearly demonstrate that selective reinforcement of wing surfaces, using judiciously placed filamentary composites, promises sizeable mass savings in the design of advanced aircraft structures.

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Equations of Motion for Flexible Cables

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Introduction

THE literature on the equations of motion of flexible cables (used here to include strings, ropes, etc.) has a long history beginning with Lord Rayleigh in his treatise on the "Theory of Sound."[1] Since that time, the solution of many problems involving towing cables, mooring cables, etc. have appeared. In addition, a set of problems concerning a moving "Threadline" have been examined by various authors[2–4]. In this note a set of equations are presented which describe a more general problem of cable motion. The general formulation contain as special cases many of the previous problems considered.

Description of the problem

Given an inextensible cable and a fixed point P, in space, one wishes to deploy the cable from another point Q in space so that the free end of the cable attaches itself to the fixed point. In particular, Q may be moving in space and one wishes to prescribe the motion of point Q so that the cable reaches the fixed point P. The cable is capable of moving in space and is acted on by gravity and fluid dynamic forces. A portion of the cable emanating from Q is shown in Fig. 1. A generic point on the cable is denoted by

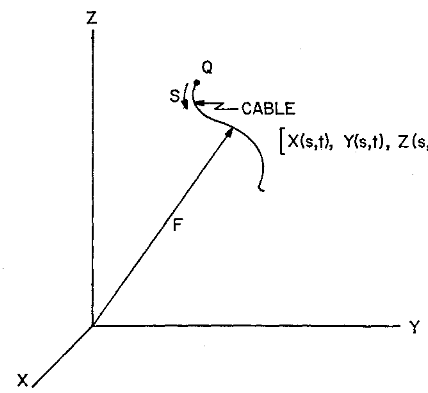

Fig. 1 Cable geometry.

$$x = x(s, t),\ y = y(s, t),\ z = z(s, t) \tag{1}$$

where s is an arc length with respect to Q and t is time. Here x, y, z are coordinates of points on the cables measured in a fixed coordinate system. The equations of motion for the cable is

$$(d/dt)(\rho\bar{q}) = \bar{F} \tag{2}$$

where ρ is the mass per unit length of the cable, $\bar{F}$ is the force per unit length, and $\bar{q}$ is the velocity vector with components V_x, V_y, V_z and

$$\bar{q} = \bar{q}[x(s,t), y(s,t), z(s,t)] \tag{3}$$

The velocity in the x direction for a point on the cable is given by

$$V_x = (dx/dt) = (\partial x/\partial s)(ds/dt) + (\partial x/\partial t) = V(\partial x/\partial s) + (\partial x/dt) \tag{4}$$

where $V = ds/dt$ is the tangential velocity of the cable. In a similar manner, we have the velocity in the y and z directions given, respectively, as

$$V_y = V(\partial y/\partial s) + (\partial y/\partial t),\ V_z = V(\partial z/\partial s) + (\partial z/\partial t) \tag{5}$$

Substituting Eqs. (4) and (5) into Eq. (2) and using the total time rate of change operator $(d/dt) = V(\partial/\partial s) + (\partial/\partial t)$ the equations of motion become

$$(\partial/\partial s)[\rho\bar{q}]V + (\partial/\partial t)[\rho\bar{q}] = (\partial/\partial s)[T(\partial\bar{r}/\partial s)] + \bar{f} \tag{6}$$

where $\bar{r} = [x(s,t),\ y(s,t),\ z(s,t)]$ and the forces have been separated into those forces due to the tension T, and the forces due to gravity and fluid dynamics, $\bar{f}$.

To complete the description of motion a continuity equation is needed to account for the change in mass. If a small control volume of cable is considered the rate of mass flow into a region is $\rho V|_s$ and the rate of mass flow out is $\rho V|_{s+\Delta s}$. The rate of change of the total mass within the region is $(\partial\rho/\partial t)\Delta s$. Equating these two and allowing $\Delta s \to 0$ results in

$$(\partial/\partial s)(\rho V) + (\partial\rho/\partial t) = 0$$

or

$$d\rho/dt + \rho\partial V/\partial s = 0 \tag{7}$$

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